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From the angle  $A$  cut *any* triangle  $XAY$  equal to the given area. Construct the triangle  $ZZ'$  where  $Z$  and  $Z'$  lie on the sides of the angle  $A$ , and such that  $AZ=AZ'=\sqrt{AX \cdot AY}$ . Then the area of this triangle is also equal to the given area. All lines cutting from the angle  $A$  a triangle having this area are tangent to an hyperbola having  $A$  as center and the sides of the angle as asymptotes.  $ZZ'$  is the tangent at a vertex. The circle with  $A$  as center and tangent to  $ZZ'$  is the auxiliary circle. It will cut the sides of the angle  $A$  in points which we will call  $M, N$ . At these points erect perpendiculars to the sides of the angle; these will intersect in the corresponding focus of the hyperbola  $S$ , say. Construct the circle with  $PS$  as diameter, and if this intersects the auxiliary circle, call one point of intersection  $Q$ . The angle  $\angle SQP$  is right, and hence  $PQ$  is a tangent to the hyperbola, and so cuts off from the angle  $A$  the required area. There is no solution when there is no point  $Q$ , nor, according to the limitations of the question, when the segment  $PQ$  intersects the segment  $BC$ .

We can treat the angles  $B$  and  $C$  in like manner.

Also solved by Elmer Schuyler, and A. H. Holmes.

231. Proposed by B. F. FINKEL, A. M., Drury College, Springfield, Mo.

A man starts from the vertex,  $A$ , of a right isosceles triangle  $ABC$ , right-angled at  $A$ , and walks to  $D$ , the middle point of  $BC$ ; from  $D$  to  $E$ , the middle point of  $AC$ ; from  $E$  to  $F$ , the middle point of  $AD$ ; from  $F$  to  $G$ , the middle point of  $DE$ ; from  $G$  to  $H$ , the middle point of  $EF$ ; from  $H$  to  $I$ , the middle point of  $FG$ ; from  $I$  to  $J$ , the middle point of  $HI$ ; and so on *ad infinitum*. Find the coördinates of his limiting position. [Suggested by Dr. Crawley].

I. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

From  $J$  he walks to  $K$ , the mid-point of  $HI$ . He then performs the same journeys in the triangle  $KIJ$  as in  $ABC$ , and so on. The sides of  $KIJ$  are  $\frac{1}{16}$  the length of the sides of  $ABC$ . Taking  $A$  as origin, the coördinates of  $K$  are  $\frac{9}{16}AD$ , and  $\frac{3}{16}DC = \frac{3}{16}AD$ . Hence the coördinates of his limiting position are

$$\frac{9}{16}AD(1 + \frac{1}{16} + \frac{1}{16^2} + \frac{1}{16^3} + \frac{1}{16^4} + \dots) = \frac{9}{15}AD = \frac{3}{5}AD,$$

$$\frac{3}{16}AD(1 + \frac{1}{16} + \frac{1}{16^2} + \frac{1}{16^3} + \frac{1}{16^4} + \dots) = \frac{3}{15}AD = \frac{1}{5}AD.*$$

II. Solution by A. H. HOLMES, Brunswick, Maine.

Draw the lines  $AD, DE, EF, FG, GH, HI, IJ, JK$ , and  $KL$ , according to the directions of the problem. Join  $AG$  and this line will pass through the point  $K$ . This is easily seen by drawing  $GN$  parallel to  $DC$ , meeting  $AD$  in  $N$ , and  $KM$  parallel to  $DC$ , meeting  $AD$  in  $M$ , and considering the similar triangles

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\*Taking  $D$  as the origin,  $DC$  and  $DA$  as the axes, the coordinates of the limiting point become  $\frac{3}{5}AD$ ,  $\frac{1}{5}AD$ . This result agrees thus with the one in Solution II by Mr. Holmes. ED.

$AKM$  and  $AGN$ .\* Similarly, a line from  $H$  to  $D$  will pass through the point  $L$ .

The point  $P$ , where  $AG$  and  $HD$  intersect is the limiting position. For every triangle within  $KIJ$  whose homologous sides are parallel to those of  $KIJ$  will have the same relation to  $KIJ$  that the latter has to  $ABC$ . Therefore the line  $AG$  passes through the right angled vertices of all such triangles within  $KIJ$ , and  $HD$  passes through the middle points of all the bases.

Let  $D$  be the origin of coördinates,  $DC$  the axis of  $x$  and  $DA$  the axis of  $y$ , then the coördinates of  $P$  are easily found to be  $x = \frac{1}{5}DC$ ,  $y = \frac{2}{5}DC$ .

Also solved by L. E. Newcomb, Elmer Schuyler, F. D. Posey, and G. W. Greenwood.

232. Proposed by O. VEBLEN, Ph. D., The University of Chicago.

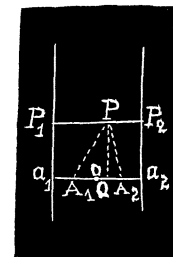
Given two parallel lines  $a_1, a_2$ , and two points  $A_1, A_2$ , upon a common perpendicular to  $a_1, a_2$  such that  $A_1$  is at the same distance from  $a_1$  as  $A_2$  is from  $a_2$ . Let  $P_1$  be the foot of the perpendicular from a point  $P$  of the same plane to the line  $a_1$  and  $P_2$  the foot of the perpendicular from  $P$  to  $a_2$ . Find the locus of  $P$  when  $\frac{PA_1}{PP_1} = \frac{PA_2}{PP_2}$ .

Solution by J. SCHEFFER, Hagerstown, Md., and A. H. HOLMES, Brunswick, Maine.

Choosing  $a_1, a_2$ , a common perpendicular to the lines  $a_1, a_2$  for the axis of  $x$ , its middle point  $O$  for the origin of orthogonal coördinates, so that  $OQ = x$ ,  $PQ = y$ , and denoting  $Oa_1 = Oa_2$  by  $a$ , and  $OA_1 = OA_2$  by  $b$ , we have  $PA_1 = \sqrt{[y^2 + (b+x)^2]}$ ,  $PP_1 = a+x$ ,  $PA_2 = \sqrt{[y^2 + (b-x)^2]}$ ,  $PP_2 = a-x$ .

From the condition of the problem

$$\frac{1}{a+x} \sqrt{[y^2 + (b+x)^2]} = \frac{1}{a-x} \sqrt{[y^2 + (b-x)^2]}.$$



Squaring, clearing of fractions, and simplifying, we finally and without difficulty obtain the equation

$$\frac{y^2}{b(a-b)} + \frac{r^2}{ab} = 1.$$

If  $a > b$ , that is, for the case that  $A_1$  and  $A_2$  are situated within the parallels  $a_1$  and  $a_2$ , the equation is that of an ellipse, whose foci are  $A_1$  and  $A_2$ , semi-axes  $\sqrt{ab}$ , and  $\sqrt{b(a-b)}$ .

If  $a < b$ , that is, for the case that  $A_1$  and  $A_2$  lie outside of the parallels  $a_1$  and  $a_2$ , the curve is an hyperbola.

Also solved by G. B. M. Zerr, L. E. Newcomb, and G. W. Greenwood.

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\* $AN = \frac{1}{3}AD$ ,  $NG = \frac{1}{3}AD$ ;  $AM = \frac{1}{6}AD$ ,  $MK = \frac{1}{6}AD$ ; hence  $AN/NG = AM/MK$ . Ed.